

Fast RC Circuit Simulation Using Artificial Neural Networks

Mohamed G. Kamal, Mohamed S. El-Mahallawy, Mohamed W. Fakhr, Yasser Y. Hanafy

Abstract—This paper presents a new technique for modeling an RC circuit using Artificial Neural Networks. In the proposed technique, the Artificial Neural Network is trained to predict the dominant characteristics of the full order model representing an RC Tree circuit and removing the non dominant characteristics that doesn't have much influence on the system. The prediction is based on minimizing the mean square error (cost function) between the desired parameters and the actual parameters. Different RC tree circuits of orders 8,10,12 and 20 are used and the reduced model of order 2 is tested. The results are compared to the SPICE simulation program output of the full order model.

Index Terms— Model Order Reduction, RC trees, Circuit modeling, Artificial Neural Networks, Mean square error

1 INTRODUCTION

Model Order Reduction (MOR) methods are used to model the input-output behavior of any large scale dynamical system over a certain range of operation using significantly smaller dimension matrices [1-2]. Simulation of large scale circuits requires solving of a high order differential equation at each node in the circuit which is time consuming, numerically demanding and heavily CPU intensive especially for very large scale circuits. So, searching for a reduced order model which preserves the dominant characteristics of the full order model and reduces the analysis time will be a demanding solution for this problem. The role of this reduced order model is to preserve the dominant characteristics of the system response of the model and reduces the simulation time and CPU usage[3]. Moreover, MOR offers an excellent route to computing the input-output response eliminating a large number of features that do not have a significant influence on the system output.

In order to obtain a reduced order model, retain the critical frequencies and minimize the mean square error, several methods of MOR have been introduced. Each of these methods has its advantages and disadvantages. Moreover, There is no method that gives the best results for all of the systems. Therefore, each system uses the best method with respect to its application. Now, some of these methods are introduced below.

Several Methods have been introduced to obtain the de

sired reduced order model. In Seventies, the Pade approximation method was introduced [4]. BaniHani and De [5] compared some Model order reduction methods for fast simulation of soft tissue response using the point collocation- based method of finite spheres. G. Parmar et al. [6] presented a model order reduction using genetic algorithm for unit impulse input and measured the integral square error and impulse response energy. C.B. Vishwakarma and R. Prasad [7] proposed a method that approximates the numerator polynomials using Pade approximation and approximates the denominator polynomials using some clustering techniques, this method ensures the stability of the system. Alsmadi et al. [8] proposed a method for MOR of dynamical systems based on Artificial Neural Networks (ANN) transformation along with the linear matrix inequality (LMI) optimization method. Alsmadi et al. [3] proposed a method for model order reduction using substructure preservation.

In this paper, a new technique for modeling any RC Tree circuit using ANN is presented based on minimizing the mean square error referred as cost function.

The rest of the paper is organized as follows. Section 2 introduces the transfer function representing the reduced model. In section 3, the training process of the artificial neural network is illustrated. In section 4, the results are introduced and finally in section 5, the conclusion is presented.

2 THE RC TREE CIRCUIT MODEL

The discrete time system representing any RC circuit is described by the following transfer function:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{a_0 + a_1s + a_2s^2 + \dots + a_{k-1}s^{k-1}}{b_0 + b_1s + b_2s^2 + \dots + b_k s^k} \quad (1)$$

where a_i represents the numerator coefficients, b_i represents the denominator and k is the number of coefficients, $X(s)$ is the input and $Y(s)$ is the output of the system.

The corresponding transfer function of the reduced model

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which contains the dominant poles only is written as:

$$G_r(s) = \frac{Y(s)}{X(s)} = \frac{c_0 + c_1s + c_2s^2 + \dots + c_{n-1}s^{n-1}}{d_0 + d_1s + d_2s^2 + \dots + d_ns^n} \quad (2)$$

where c_i represents the numerator coefficients, d_i represents the denominator coefficients and n is the number of the reduced coefficients.

To obtain a stable macro-model, the reduced model transfer function is converted to the pole residue model using partial fraction expansion and can be written as:

$$G_r(s) = \sum_{i=0}^n \frac{R_i}{s - P_i} \quad (3)$$

where R are the residues and P are the poles of the reduced model.

Hence, the impulse response equation of the reduced order model can be written as:

$$g_r(t) = \sum_{i=0}^n R_i e^{-\frac{t}{P_i}} \quad (4)$$

To construct a reduced model whose impulse response is very close to the original model in an efficient and fast way regardless to the full model size, an ANN is trained to estimate the impulse response of the RC Tree circuit and minimizing the mean square error between the impulse response curves of the full order model and reduced order model [3].

3 THE ARTIFICIAL NEURAL NETWORKS MODEL

In order to model any circuit, two approaches are used; the physical approach and the black box approach. Each of these approaches has its advantages and disadvantages. However, when there is no complete knowledge of the physical parameters of a device the black box approach is used. The behavior of the circuit is captured by using a set of input data and the corresponding output response data. Then an approximation is performed over a set of measured data in order to find a convenient analytical equation to use it in the simulation of the circuit. The main advantage of the black box approach is that the simulation time is small compared to the physical approach.

Multilayer perceptron NN is one of the most common types of NN used in the simulation of non linear functions. It is used to implement nonlinear transformations for function approximations[9]. The network is composed of a set of source nodes that represent the input layer, one or more hidden layers and an output layer. Each layer computes the activation function of the weighted sum of the inputs. The input signal propagates through the layers in a forward direction from the input layer to the output layer (and passes through all of the hidden layers layer by layer)[10]. The input-output relationship between of each node of the hidden layer is as follows[11]:

$$y = f(\sum_i w_i x_i + b) \quad (5)$$

where w_i is the weight that connects the i^{th} node and the current node, x_i is the output from i^{th} node of the previous layer, b is the threshold value of the current node and f is the activation function. The outputs of the hidden layers are distributed over the next layer until the last one where the outputs are fed into a layer of output units.

In this paper, The ANN is trained to estimate the dominant poles and residues of any high order transfer function regardless to the values of the poles and residues of the impulse or step response representing the transfer function

The learning process is performed using The back-propagation algorithm which adjusts the free parameters (weights and biases) of the network to minimize the mean square error (MSE) referred as the cost function [3] as written below:

$$MSE = \frac{1}{N} \sum_{n=1}^N \frac{1}{2} \sum_{j \in C} (y_j(n) - z_j(n))^2 \quad (6)$$

where MSE is the cost function as a measure of learning performance, $y_j(n)$ is the desired response of the full order model, $z_j(n)$ is the output response of the reduced model, neuron j is the output node, set C includes all the neurons in the output layer of the network and N is the total number of patterns contained in the training set.

4 SIMULATION RESULTS

Considering an RC tree circuit that contains a large number of poles and residues, the ANN is used to reduce its order to the second order. The ANN is offline trained with the impulse response curves of circuits with different values of poles and residues and used online to estimate the values of the two dominant poles and residues of the reduced transfer function.

The Artificial Neural Network used contains two hidden layers and one output layer with an activation function which is tangent hyperbolic sigmoid in the hidden layers and purelinear in the output layer.

The network is trained using gradient descent algorithm with 22000 data points with a mean square error (cost function). The range of the dominant poles used is between 0 and 0.4 while the range of the residues of the dominant poles is between 0.5 and 1.

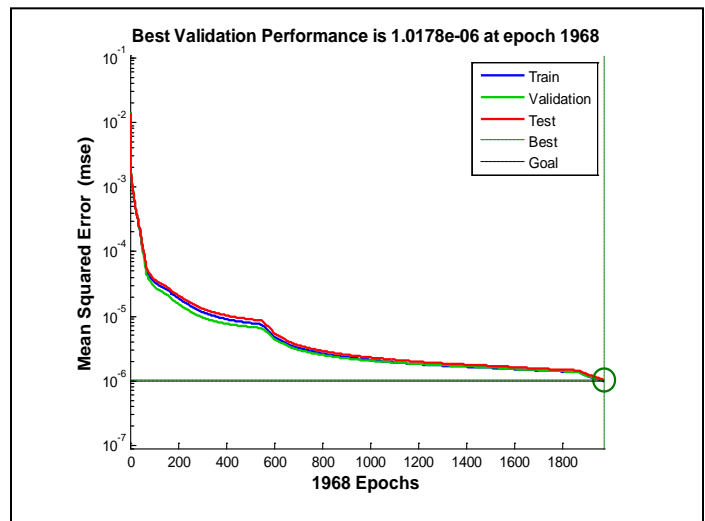


Figure 1 shows the training process of the ANN. In this figure, the mean square error between the exact (SPICE simulated) and the estimated (using ANN) is 1.0178×10^{-6} after training the ANN for 1968 epochs.

After completing the training process, the Neural Network is tested in 4 different cases using generated testing data other than that of the training data.

In the first case, we consider 22000 different test patterns of 8th order RC tree circuits; each pattern contains two dominant poles and 6 non-dominant poles. The trained ANN is used to estimate the dominant poles and residues of each pattern. The trained network succeeded to reduce the order of the transfer function of the circuit to the second order with mean square error and mean absolute error histograms shown in figures 2 and 3.

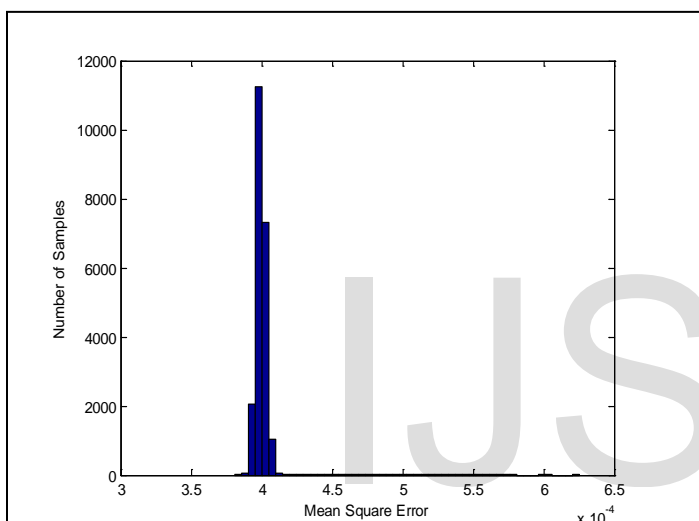


Fig. 2. Mean square error histogram (1st case)

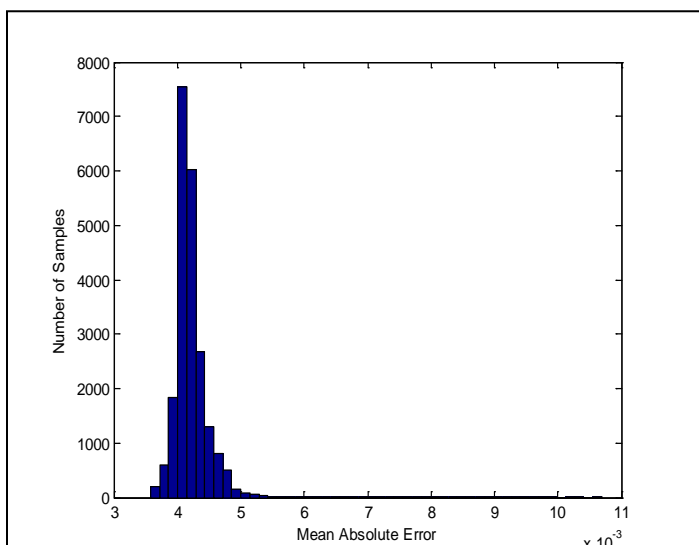


Fig. 3. Mean absolute error histogram (1st case).

It is clear that the ANN is able to estimate the dominant poles and residues of the full order model with an acceptable range of error.

Figure 4 shows an example of the exact and estimated impulse response of the tested patterns of the RC circuit trees of order 8 with 2 dominant poles and 6 non-dominant poles. The ANN succeeded to estimate the impulse response with mean square error of 4.1065×10^{-4} which is acceptable.

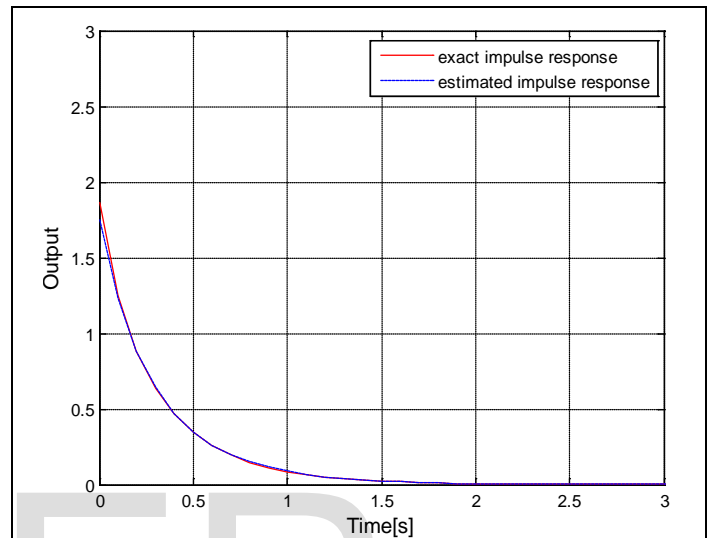


Fig. 4. Impulse response of the 8th order RC Tree (1st case).

In the second testing case, we consider another 22000 patterns of 10th order RC tree circuits impulse responses; each pattern contains two dominant poles and 8 non-dominant poles, the same trained ANN is used to estimate the dominant poles and residues of each pattern. The trained network succeeded to reduce the order of the transfer function to the second order with mean square error and mean absolute error histograms shown in figures 5 and 6.

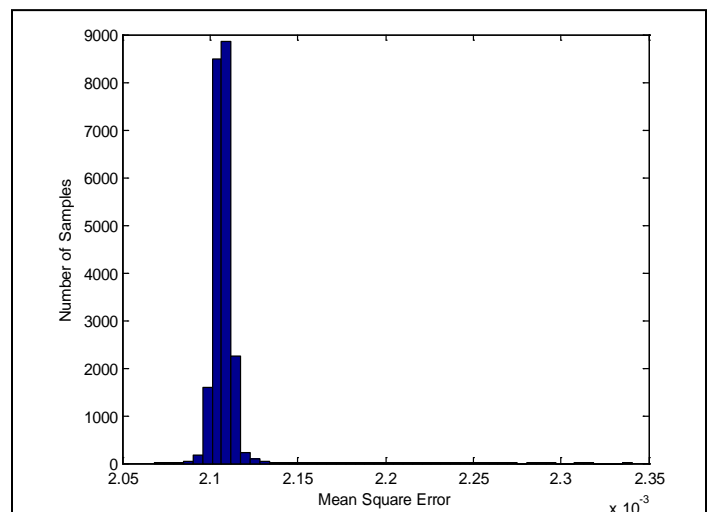
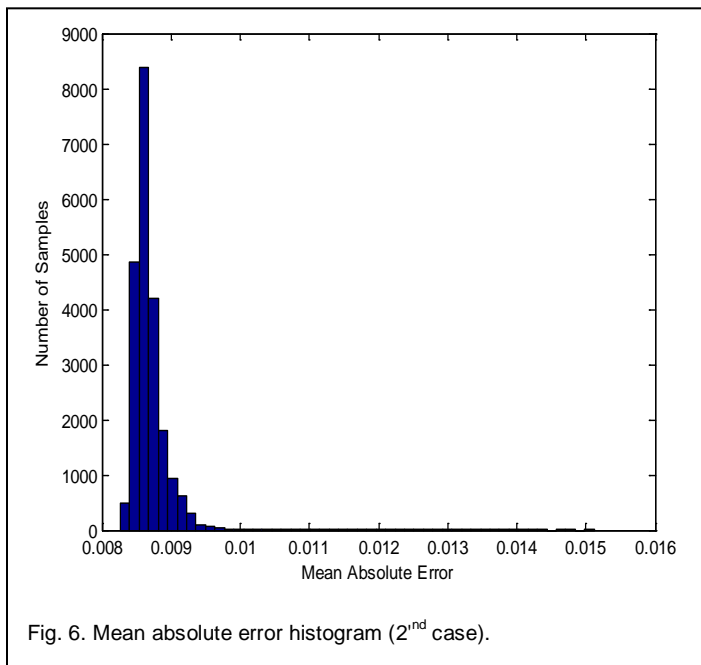


Fig. 5. Mean square error histogram (2nd case).



Figures 5 and 6 show that the ANN is still able to estimate the dominant poles and residues of the full order models and the error is approximately still the same after increasing the number of the non-dominant poles and residues

Figure 7 shows the impulse response of the tested pattern of an RC tree circuit of order 10 with the same 2 dominant poles used previously and 8 non-dominant poles. The ANN is able to estimate the impulse response with mean square error 0.0021 which is increased compared to example 1 as the number of non-dominant poles is increased but acceptable.

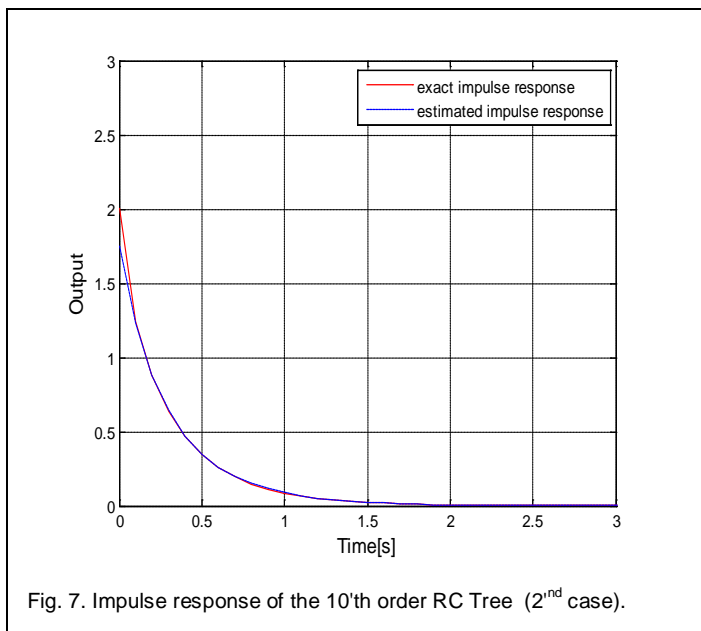


Fig. 7. Impulse response of the 10th order RC Tree (2nd case).

In case 3, we consider another 22000 different patterns of an 12th order RC tree circuits impulse responses, each pattern contains two dominant poles and 10 non-dominant poles. The same ANN is used to estimate the dominant poles and resi-

dues of each pattern. The trained network succeeded to reduce the order of the transfer function to the second order with mean square error and mean absolute error histograms shown in figures 8 and 9.

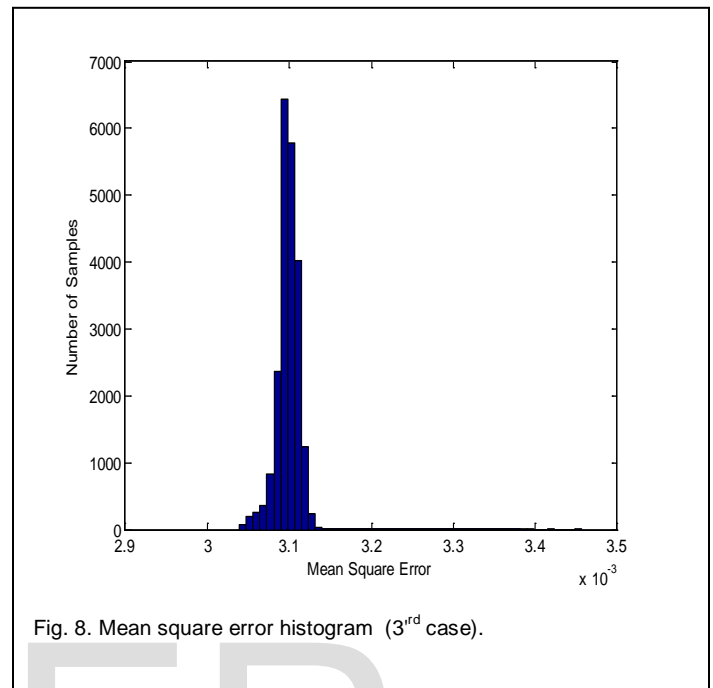


Fig. 8. Mean square error histogram (3rd case).

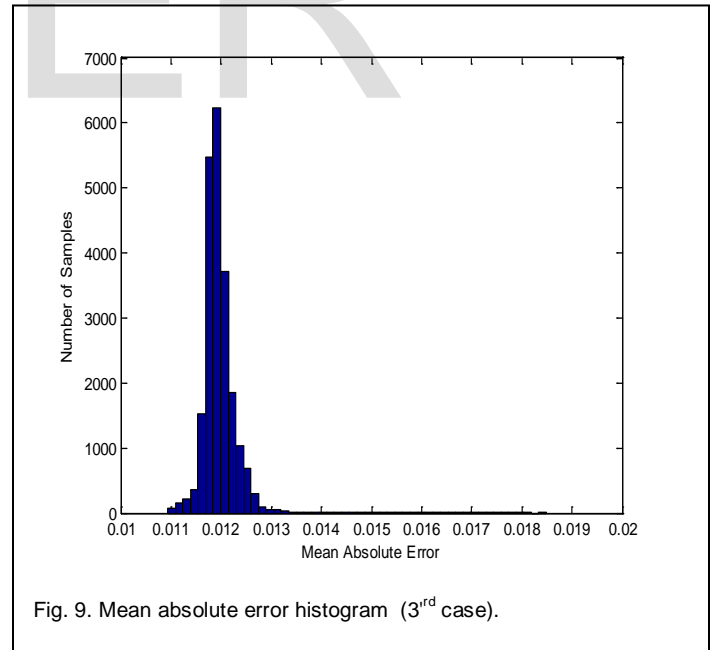
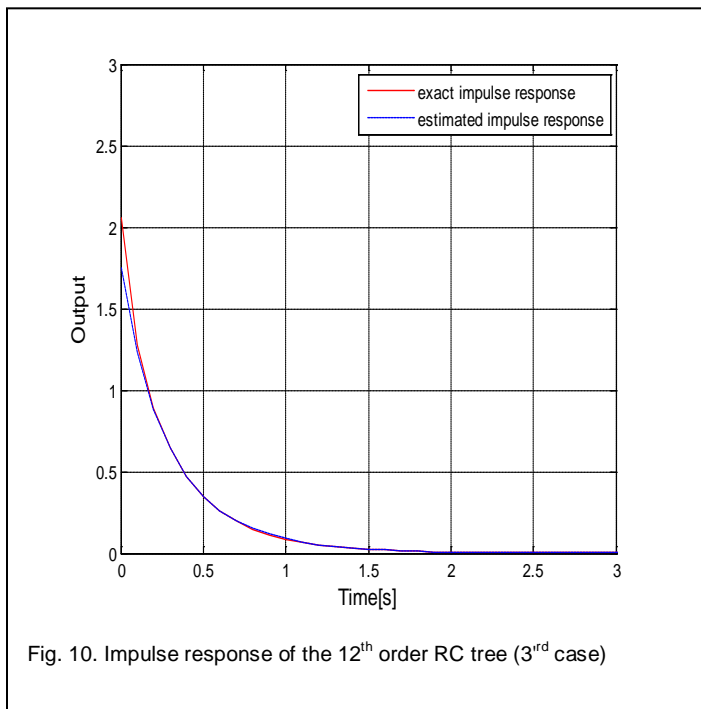


Fig. 9. Mean absolute error histogram (3rd case).

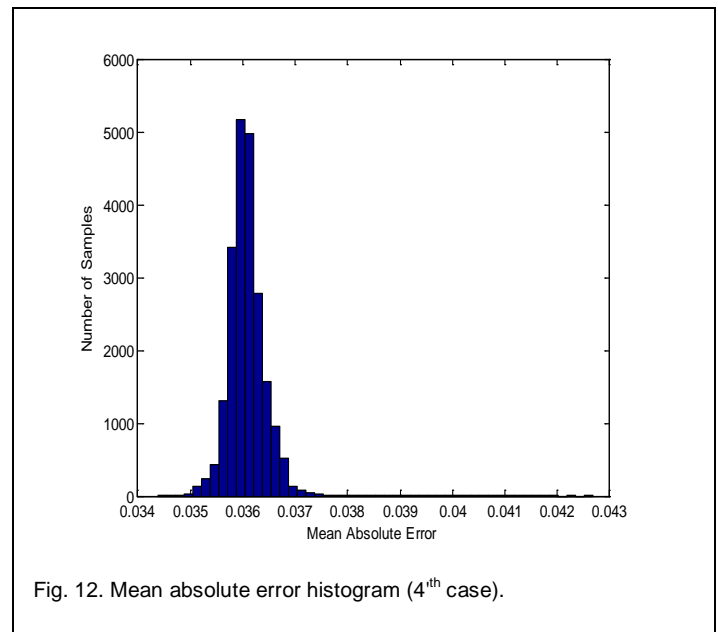
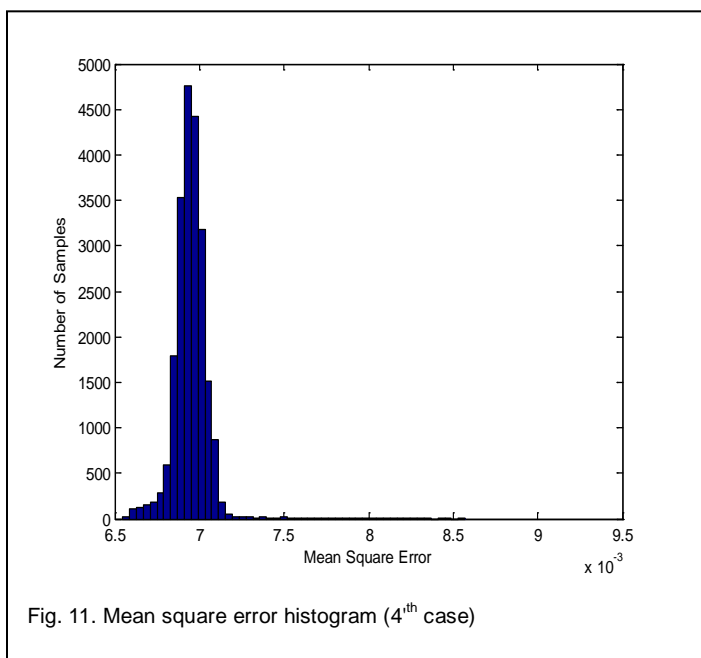
It is clear that the ANN is still able to estimate the dominant poles and residues of the full order model and the error is greater than that of the previous cases but still acceptable.

Figure 10 shows the impulse response of a tested pattern of an RC tree circuit of order 12 with 2 dominant poles and 10 non-dominant poles. The ANN succeeded to estimate the impulse response with mean square error 0.0031 with more error

than that of examples 1 and 2 but still acceptable.

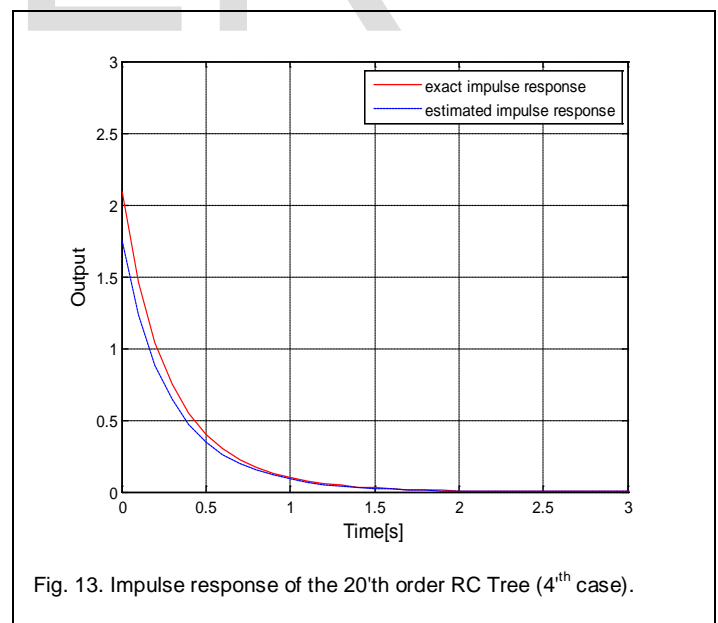


In the last case, another 22000 pattern of 20th order RC tree circuit is tested; each pattern contains two dominant poles and 18 non-dominant poles. The same ANN is used to estimate the dominant poles and residues of each pattern. The trained network succeeded to reduce the order of the transfer function to the second order with mean square error and mean absolute error histograms shown in figures 11 and 12.



It is clear that the ANN is still able to estimate the dominant poles and residues of the full order model and the error is increased compared to the previous cases due to the effect of the large number of the non-dominant poles and residues.

Figure 13 shows the impulse response of a tested pattern of RC tree circuit of order 20 with 2 dominant poles and 18 non-dominant poles. The ANN succeeded to estimate the impulse response with mean square error 0.0069 which is large compared to the previous examples as the number of poles and residues is increased.



A comparison between the values of the exact and estimated dominant poles (P1, P2) and residues (R1, R2) for all the examples mentioned previously is given in table 1.

TABLE 1

Exact and estimated dominant poles and residues of 8th, 10th, 12th and 20th order RC circuit with the same dominant poles.

Residues and poles	Exact dominant poles and residues	Estimated dominant poles and residues
R2	1	1.0102
R1	0.74	0.7382
P2	0.4	0.3997
P1	0.2	0.2003

5 CONCLUSION

A new technique is developed to estimate the poles and residues of the reduced order model of any system (transfer function representing an RC circuit) regardless to the number of poles and residues representing the RC circuit using Artificial Neural networks. This technique is obtained by training the ANN with the impulse response curves of different RC circuits to obtain a model. This model estimates the dominant characteristics of the full order model without using nodal analysis equations used by the simulation programs (e.g. SPICE) and retains them in the reduced order model that produces a very close response compared to the full order model. This technique will reduce simulation time with an acceptable accuracy compared to SPICE program.

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